Northeastern University
Khoury College of
Computer Sciences

Integrating Symmetry Into Differentiable Planning With Steerable Convolutions

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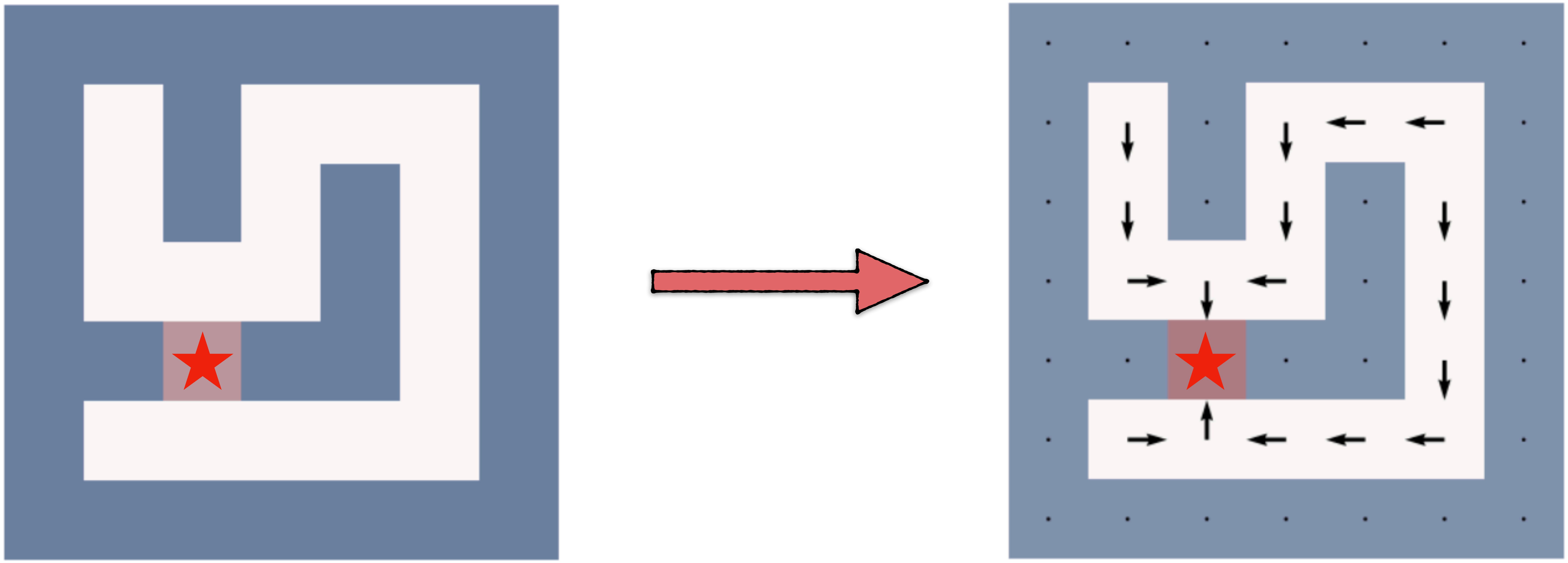
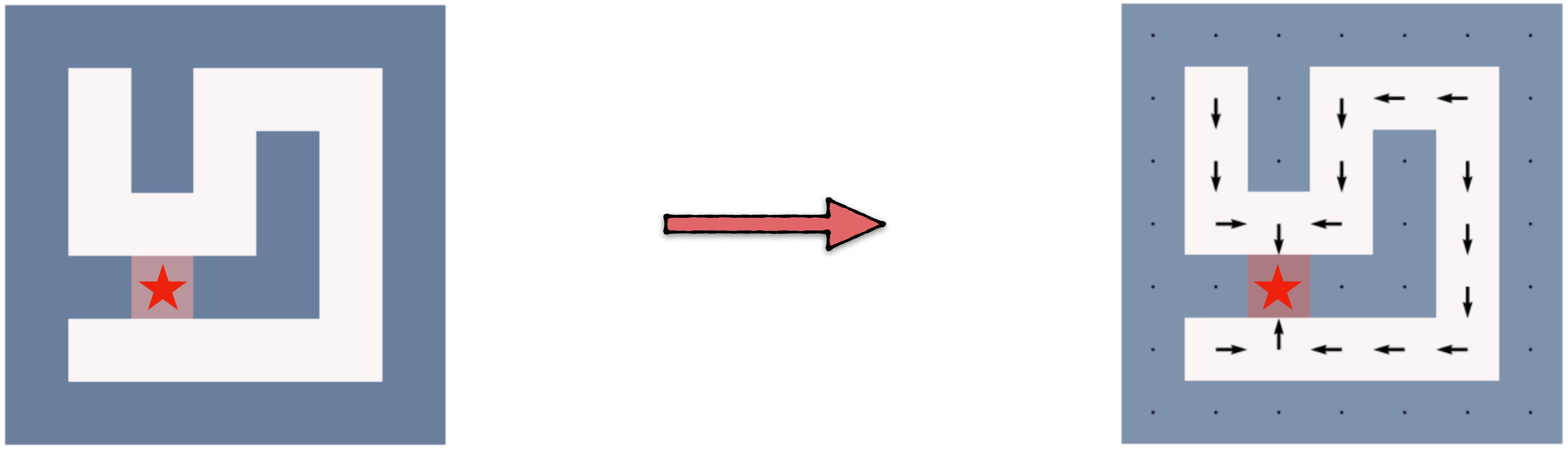
Path Planning



Find shortest path / optimal actions to the goal location (red)

Motivation

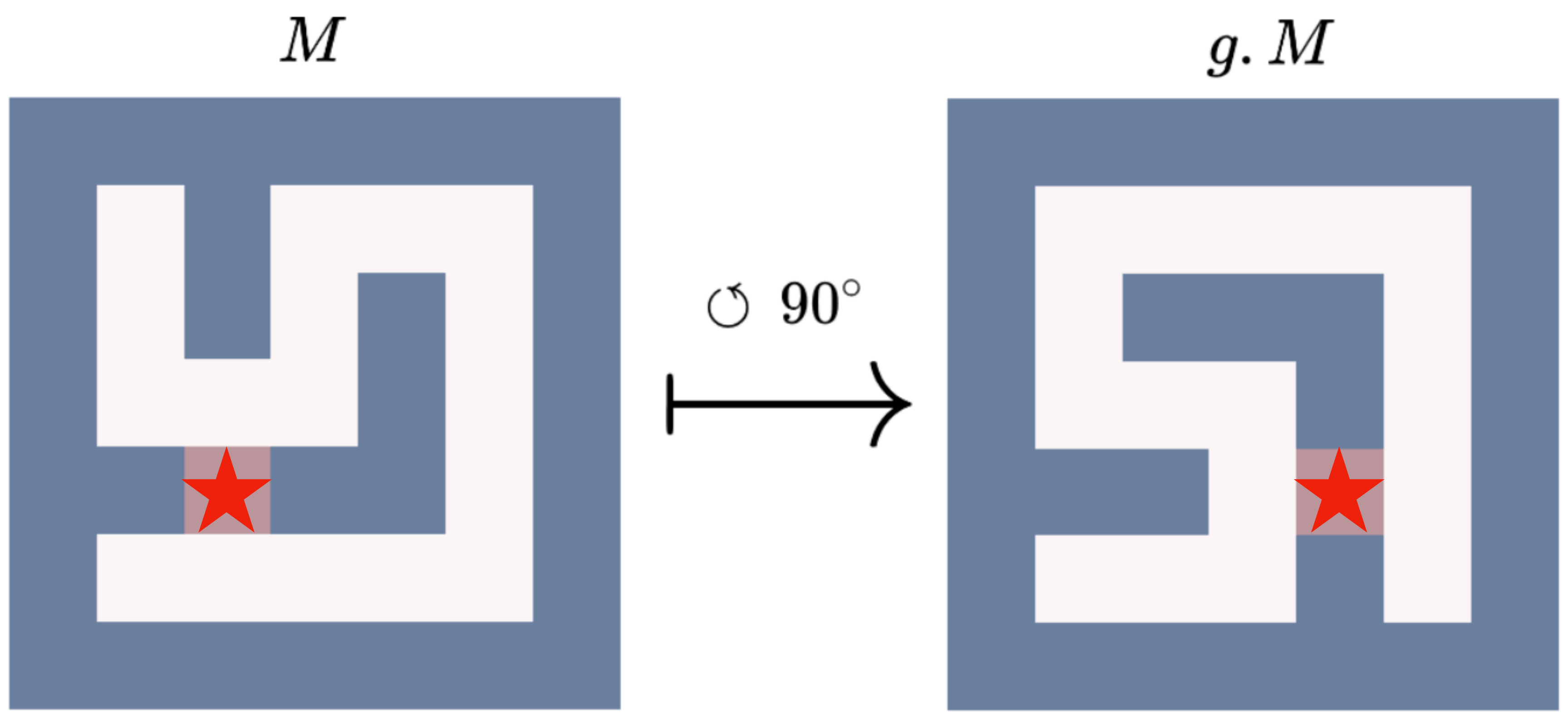


Motivation — Improve training efficiency and generalization for differentiable planning algorithms

We study a geometric structure of grid — discrete symmetry

Symmetry in Path Planning

What does the symmetry look like?



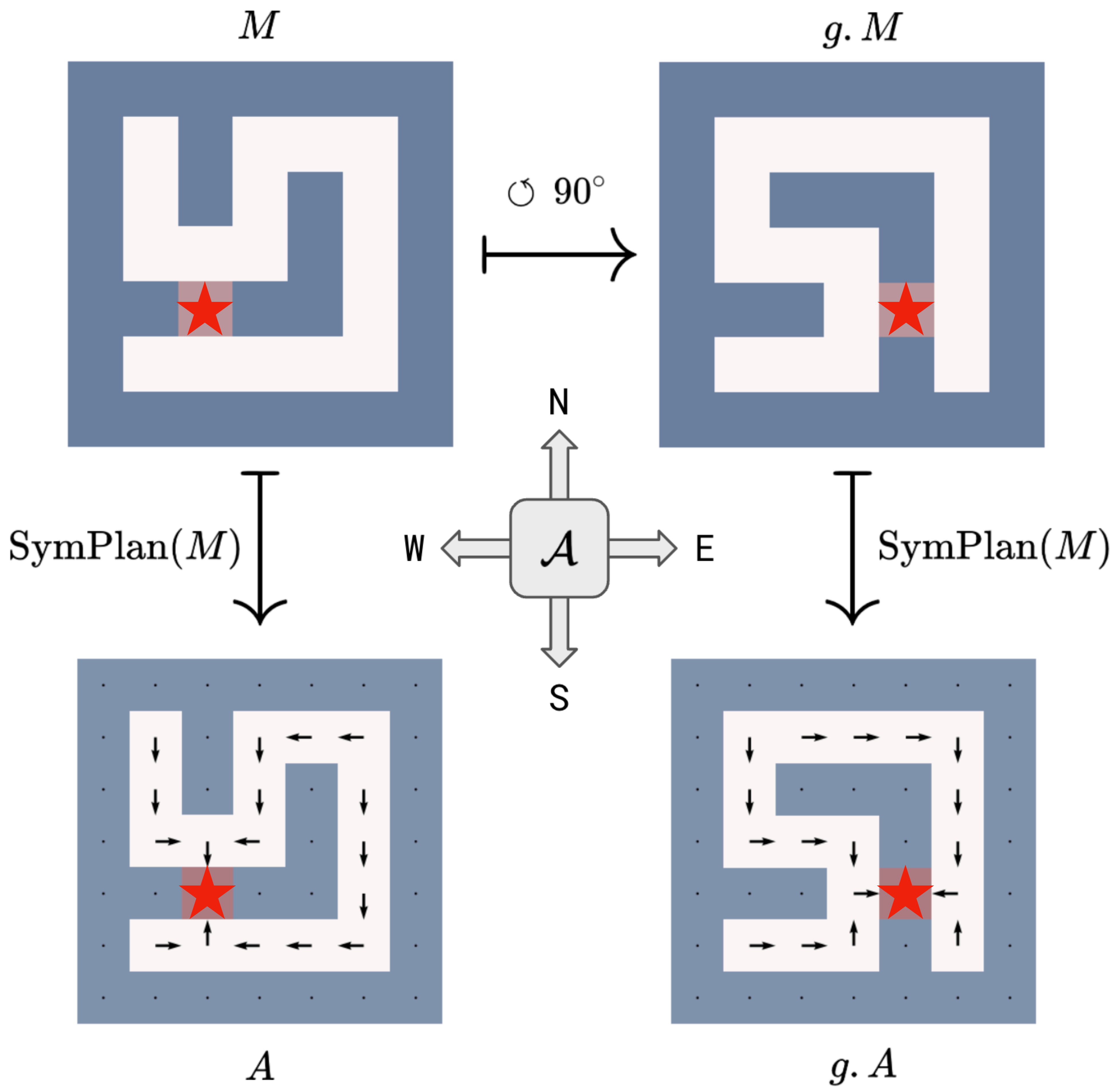
Symmetry in Path Planning

What does the symmetry look like?

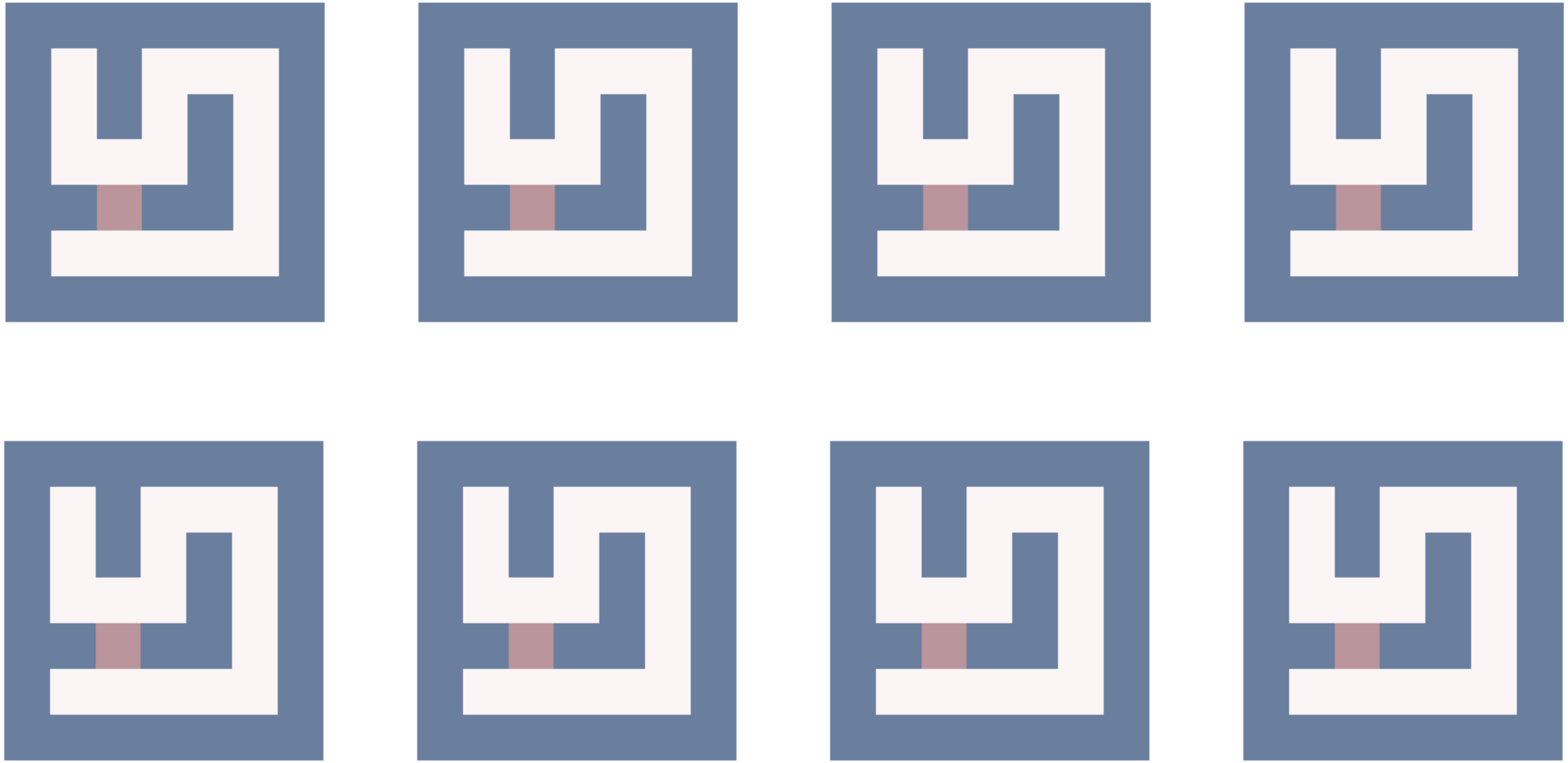
They can be described by

Equivariance

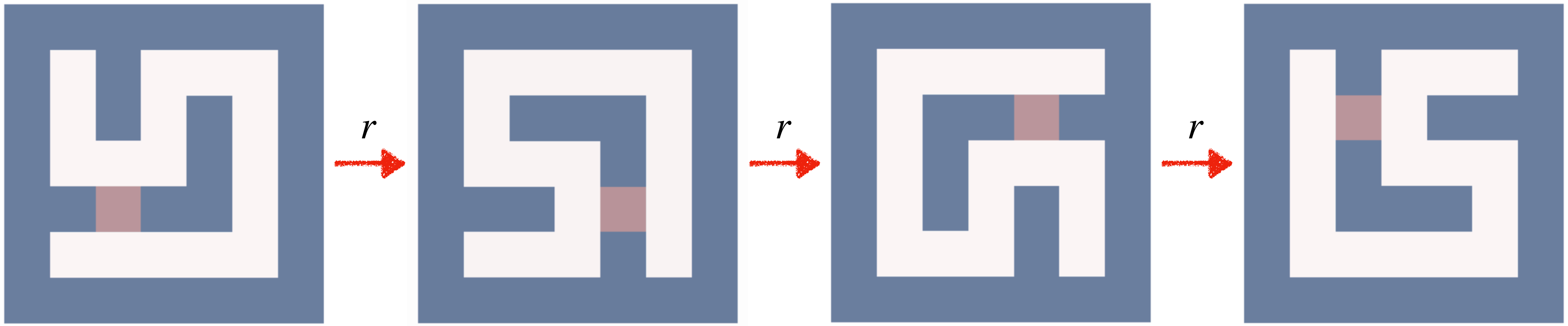
$$\circlearrowleft 90^\circ \circ \big(\text{Plan}(M)\big) = \text{Plan}(\circlearrowleft 90^\circ \circ M)$$



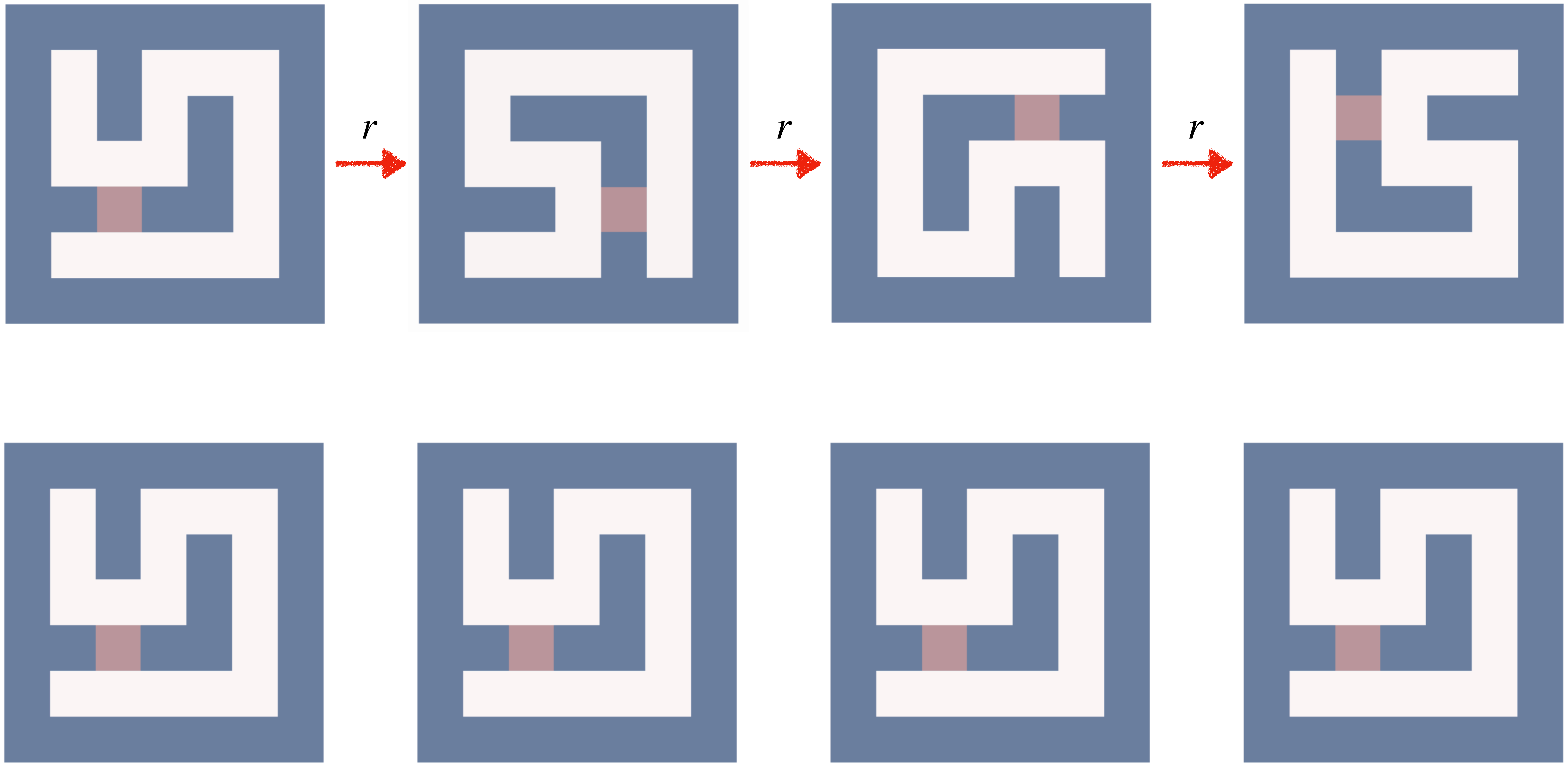
Symmetry: All Rotations and Reflections



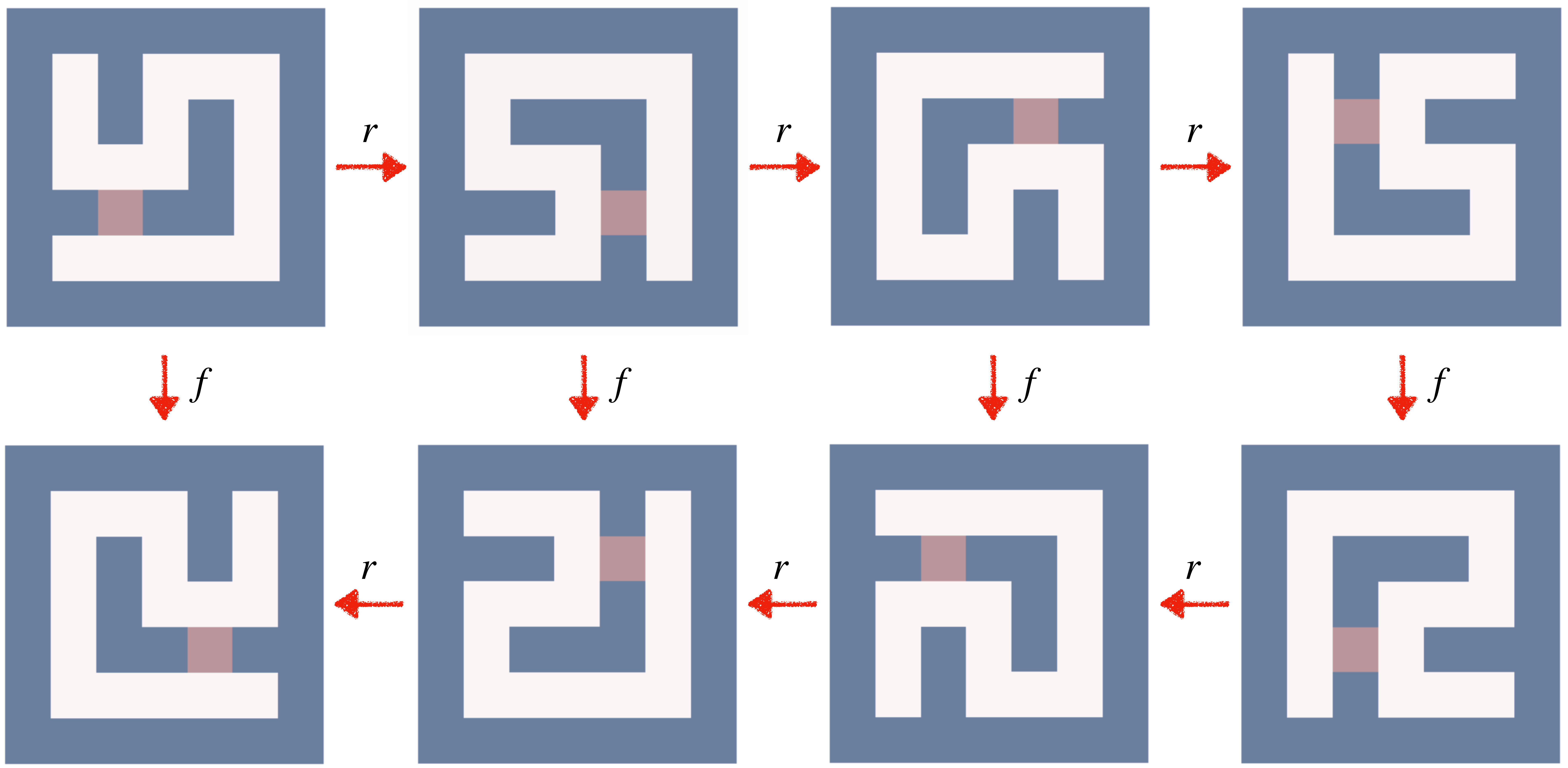
Symmetry: Rotations



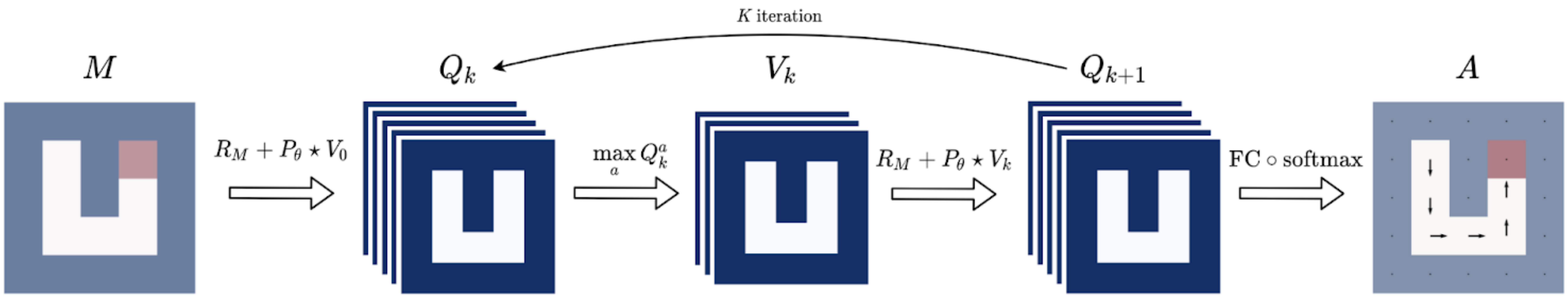
Symmetry: Rotations and Reflections



Symmetry: All 8 Transformations in D_4



Background: Value Iteration Networks



- *Value Iteration Networks* implement Value Iteration by CNNs
- It iteratively applies Bellman operator and differentiates through multiple layers

Tamar et al. Value Iteration Networks. NIPS 2016.

Value Iteration with Symmetry

Every update is equivariant
— Local Equivariance

Entire planning is equivariant
— Global Equivariance

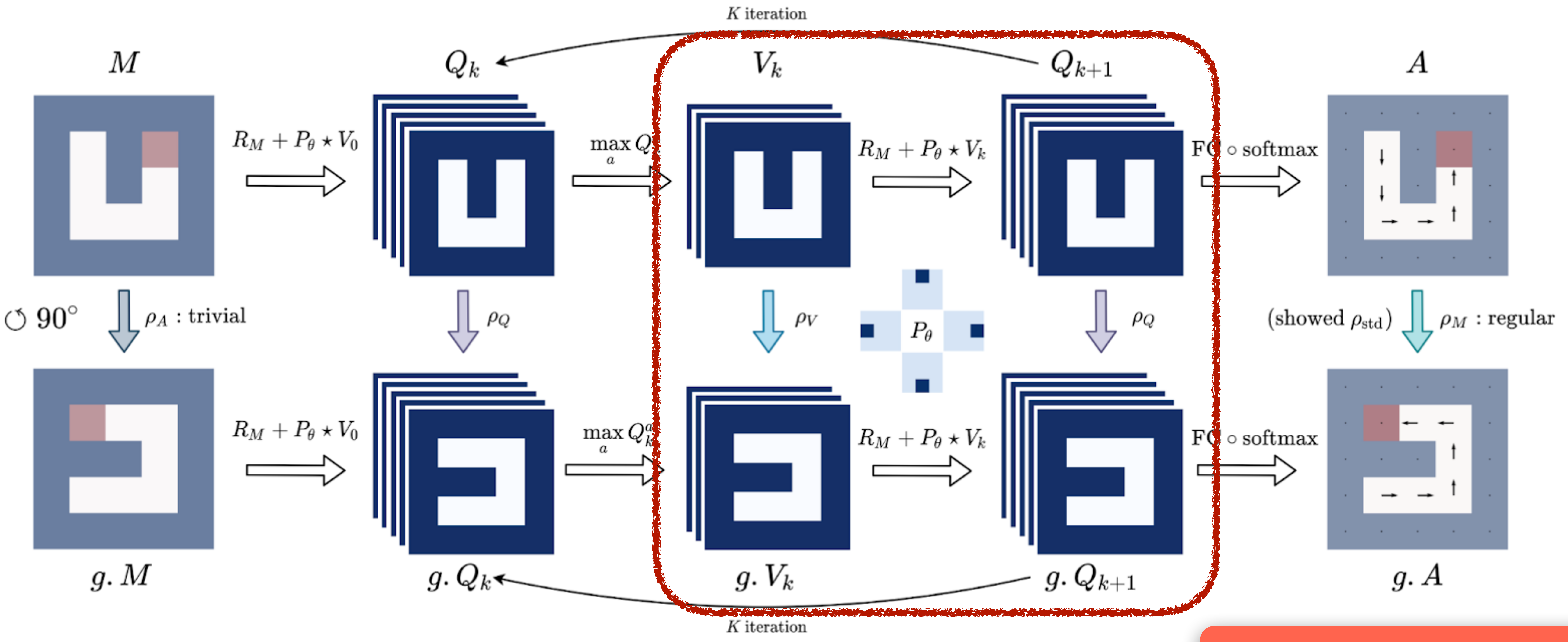
$$\circlearrowleft 90^\circ \circ \textbf{VI}(M) \equiv \circlearrowleft 90^\circ \circ \mathscr{T}^\infty[V_0] = \mathscr{T}^\infty[\circlearrowleft 90^\circ \circ V_0] \equiv \textbf{VI}(\circlearrowleft 90^\circ \circ M)$$

- Use steerable convolution, equivariant to rotation and reflection:

$$\bar{Q}^{(k)} = \bar{R}^a + \textbf{Conv2D}(\bar{V}^{(k-1)}; W^V_{\bar{a}}) \longrightarrow \bar{Q}^{(k)}_{\bar{a}} = \bar{R}_{\bar{a}} + \textbf{SteerableConv}(\bar{V}; W^V)$$

Replace

Main Pipeline: Symmetric Value Iteration Network



Every pair is equivariant

We use steerable convolutions to integrate symmetry in VINs.

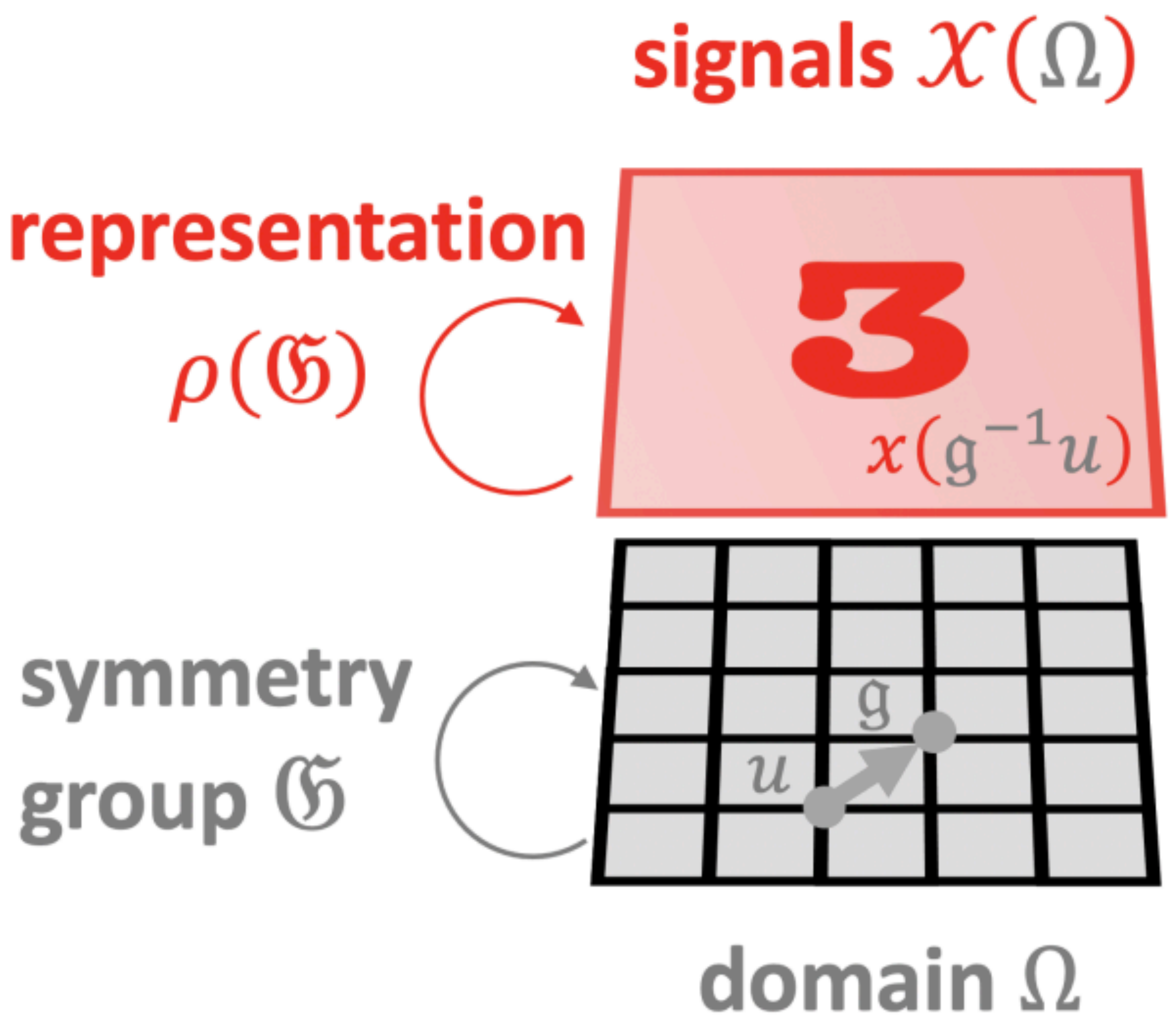
Main Pipeline: Symmetric Value Iteration Network

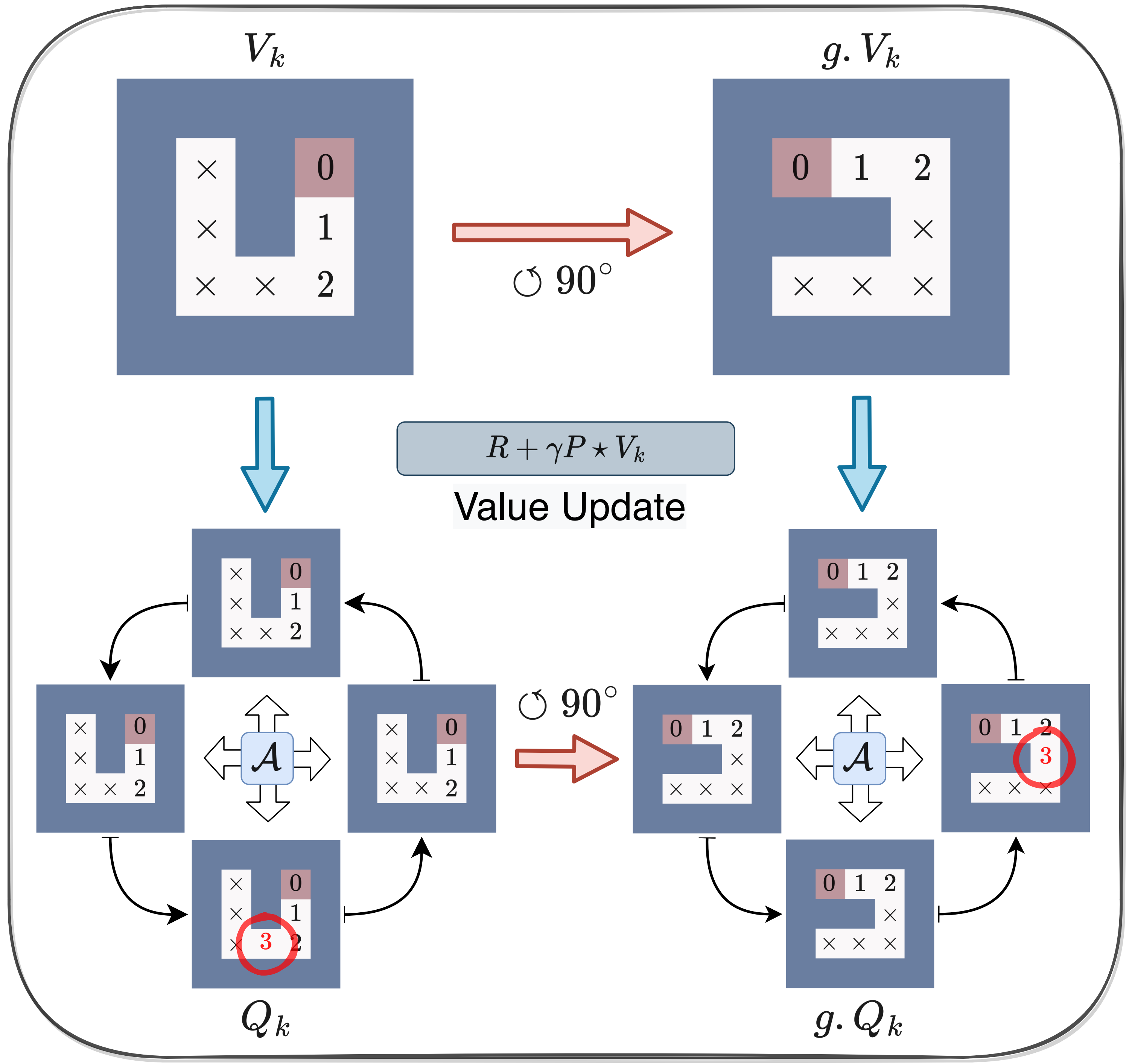
We use steerable convolutions to integrate symmetry in VINs.

Fully differentiable
— *Avoids explicitly computing the equivalence classes of symmetric states*

Backed up by theoretical insights
— *Based on geometric structure of grid — To show next*

Key Insights





1: Represent (value) functions as "fields"

2: Value iteration as convolution (network)

Bronstein et al. (2021): Geometric Deep Learning: Grids, Groups, Graphs, Geodesics, and Gauges. arXiv.

Theoretical results

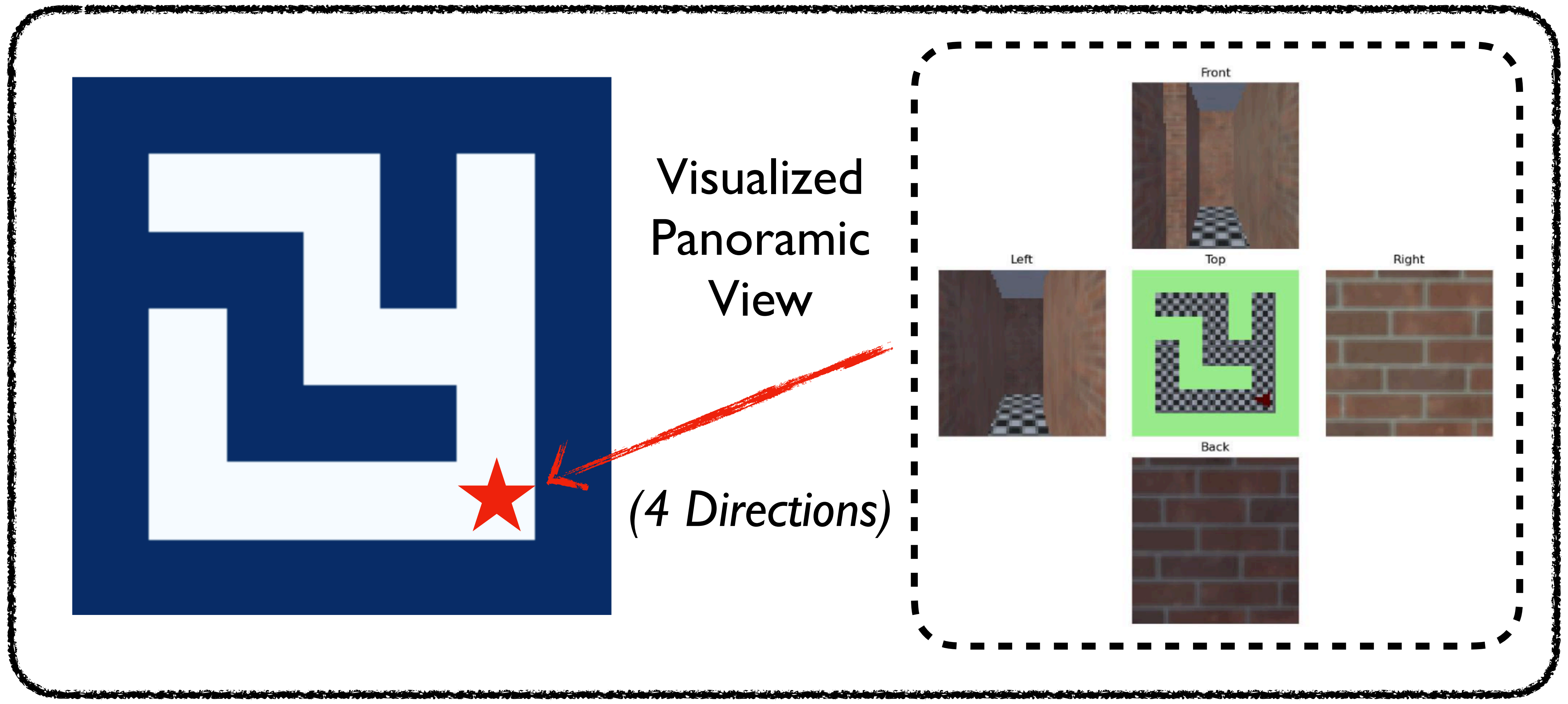
Theorem 1 (informal): **Value iteration** for **path planning*** is equivariant to translation, rotation, and reflection

Theorem 2 (informal): **Value iteration** for **path planning*** is a form of **steerable convolution network****

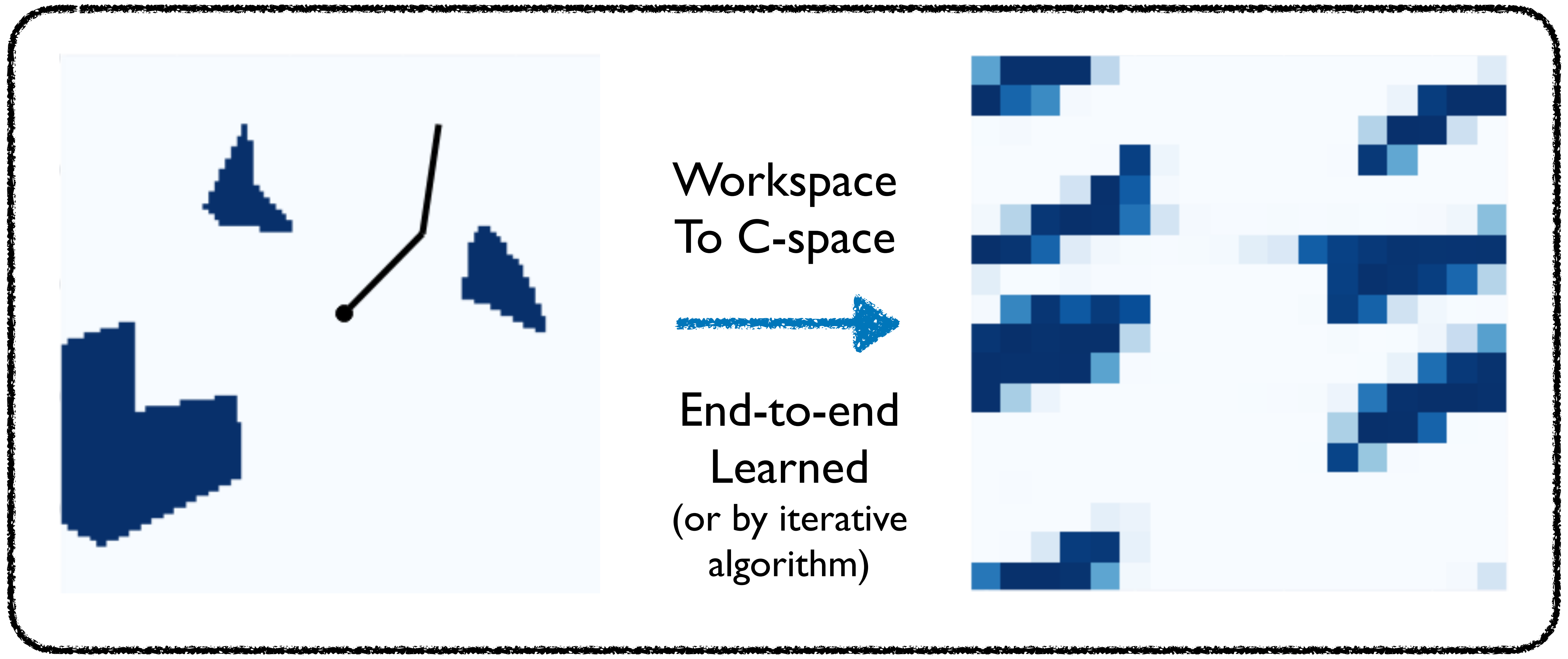
*: Path planning on 2D grid, an example of homogeneous spaces
**: Steerable CNN over grids, equivariant under induced representations

Cohen et al. (2017): Steerable CNNs, ICLR 2017

Experiment: Setup



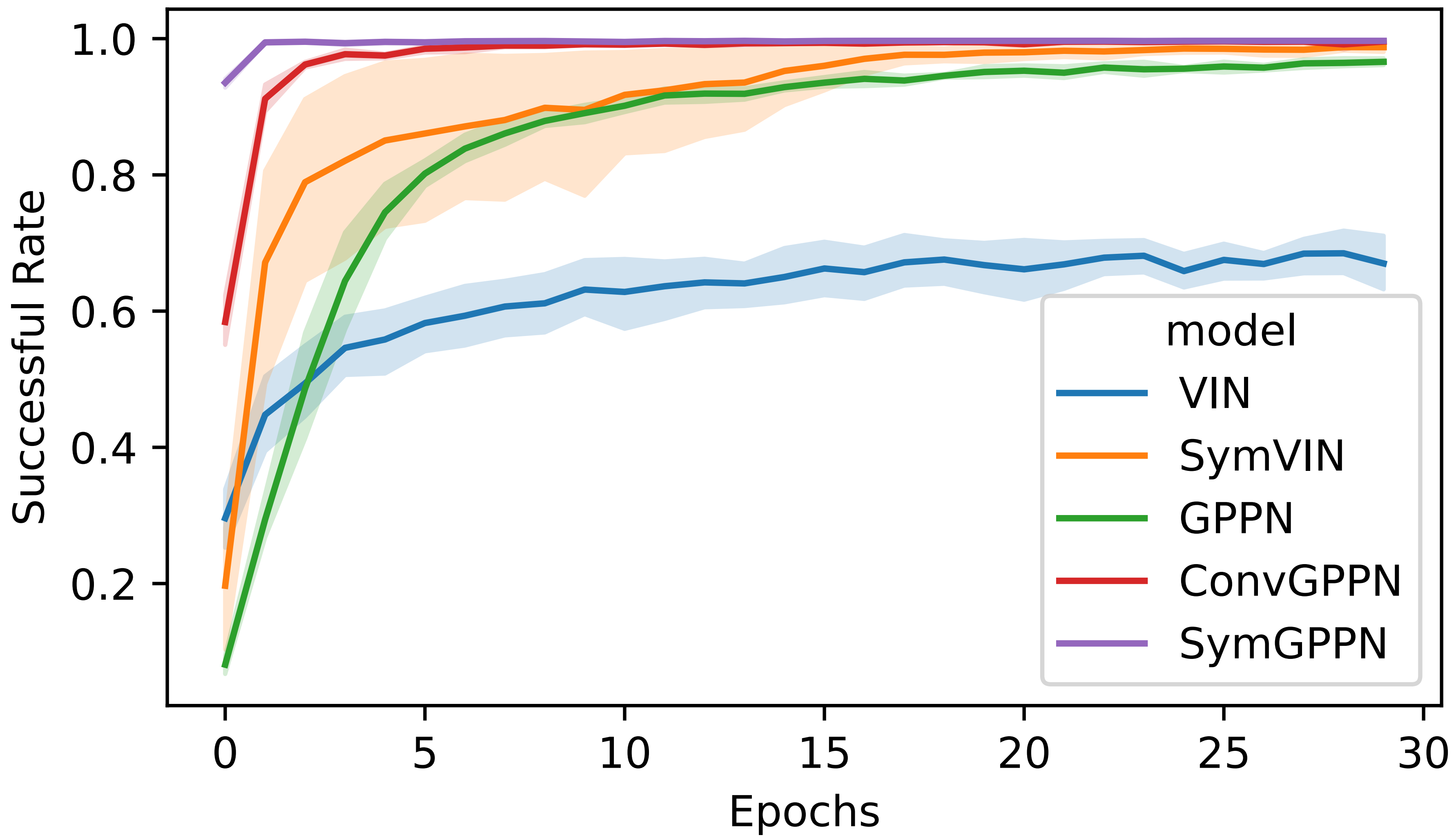
2D and Visual
Maze Navigation



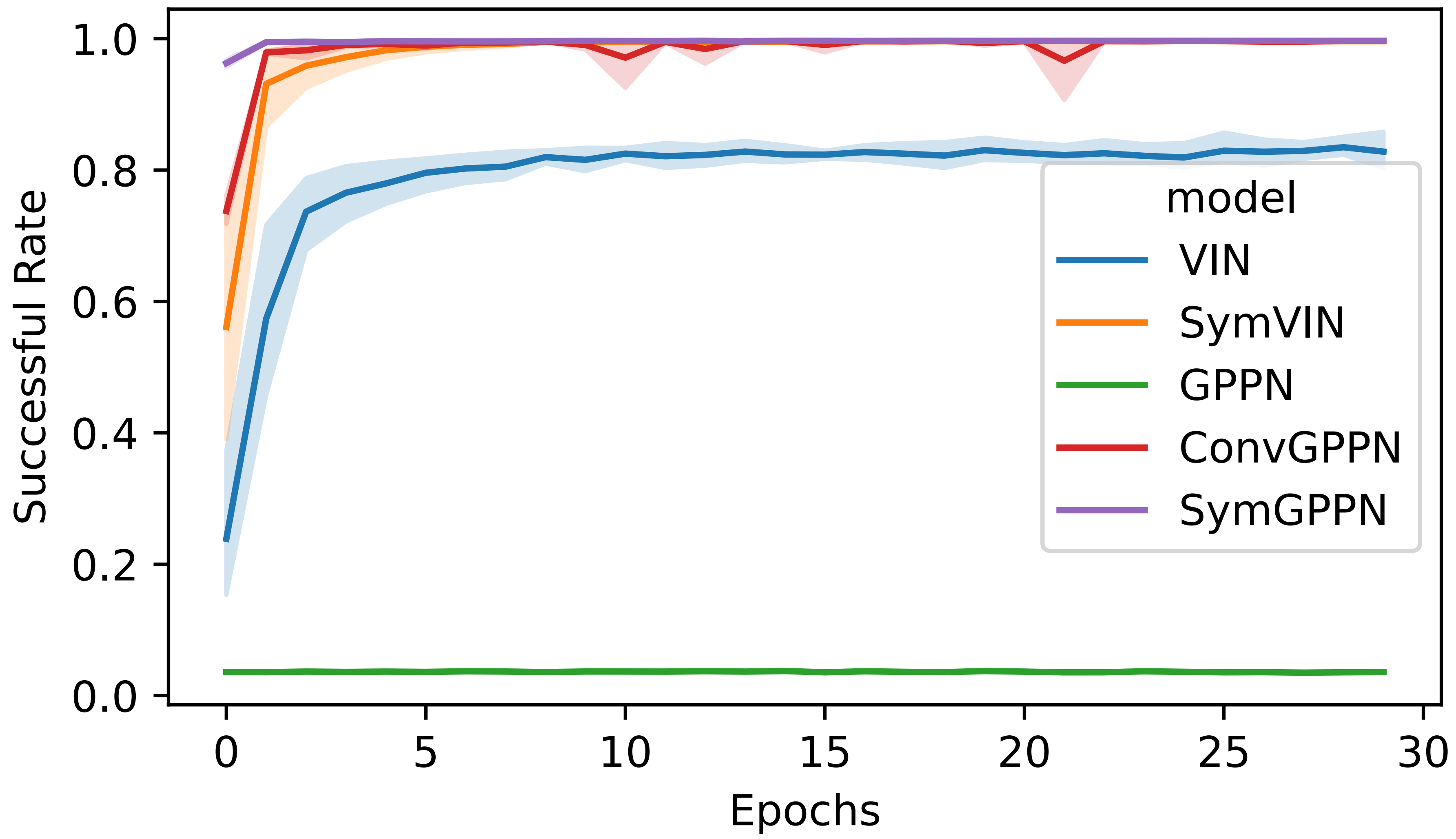
2-DOF Manipulation
In Workspace and C-space

Results: Training

2D Maze Navigation



2-DOF Manipulation



- Training curves on 15x15 maps
- All networks use 3x3 filters, fixed 30 iterations (shared convolution layers)
- SymGPPN — recurrent version of VIN with steerable convolution

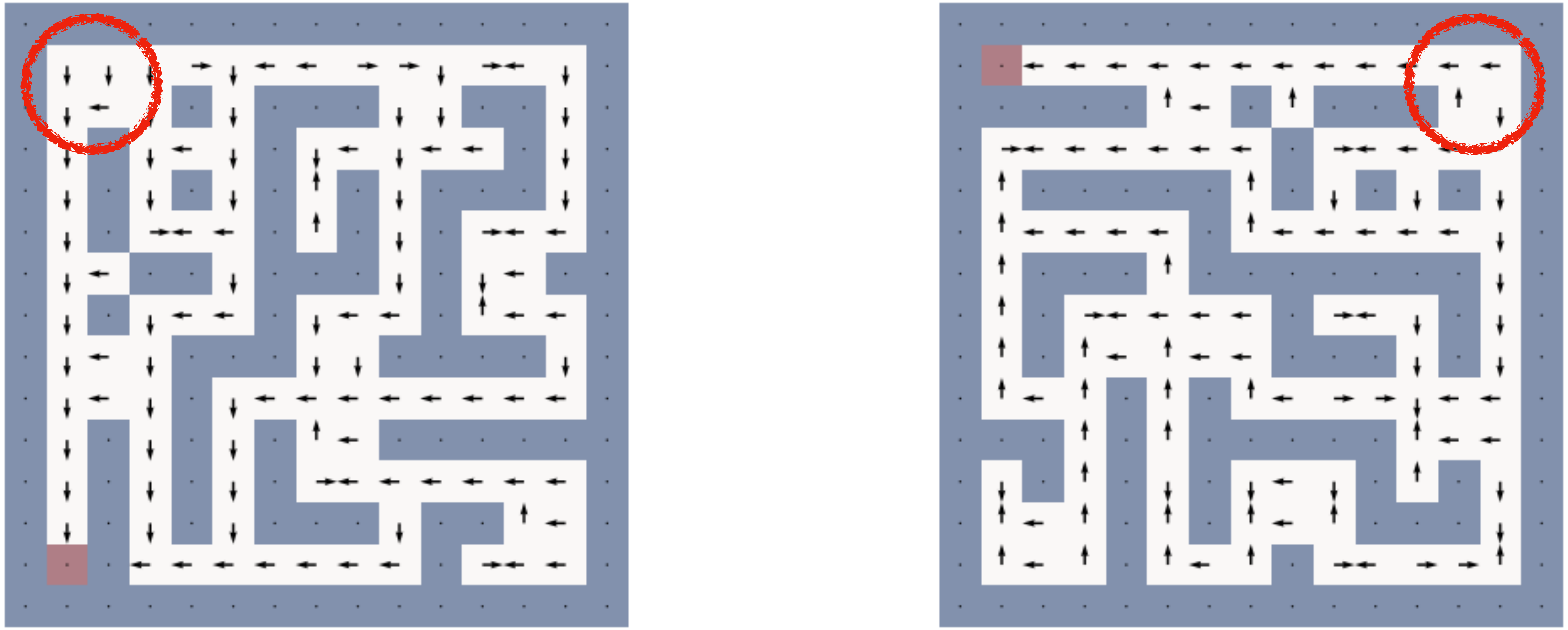
Results: Evaluation on test maps

Method (10K Data)	Navigation 15 × 15	Navigation 28 × 28	Navigation 50 × 50	Visual	Manipulation 18 × 18	Manipulation 36 × 36	Workspace
VIN	66.97	67.57	57.92	50.83	77.82	84.32	80.44
SymVIN	98.99	98.14	86.20	95.50	99.98	99.36	**91.10**
GPPN	96.36	95.77	91.84	93.13	2.62	1.68	3.67
ConvGPPN	99.75	99.09	97.21	98.55	99.98	99.95	89.88
SymGPPN	**99.98**	**99.86**	**99.49**	**99.78**	**100.00**	**99.99**	90.50

- Evaluation on all 4 tasks
- Our Symmetric Planners outperform all baselines

Visualization: VIN

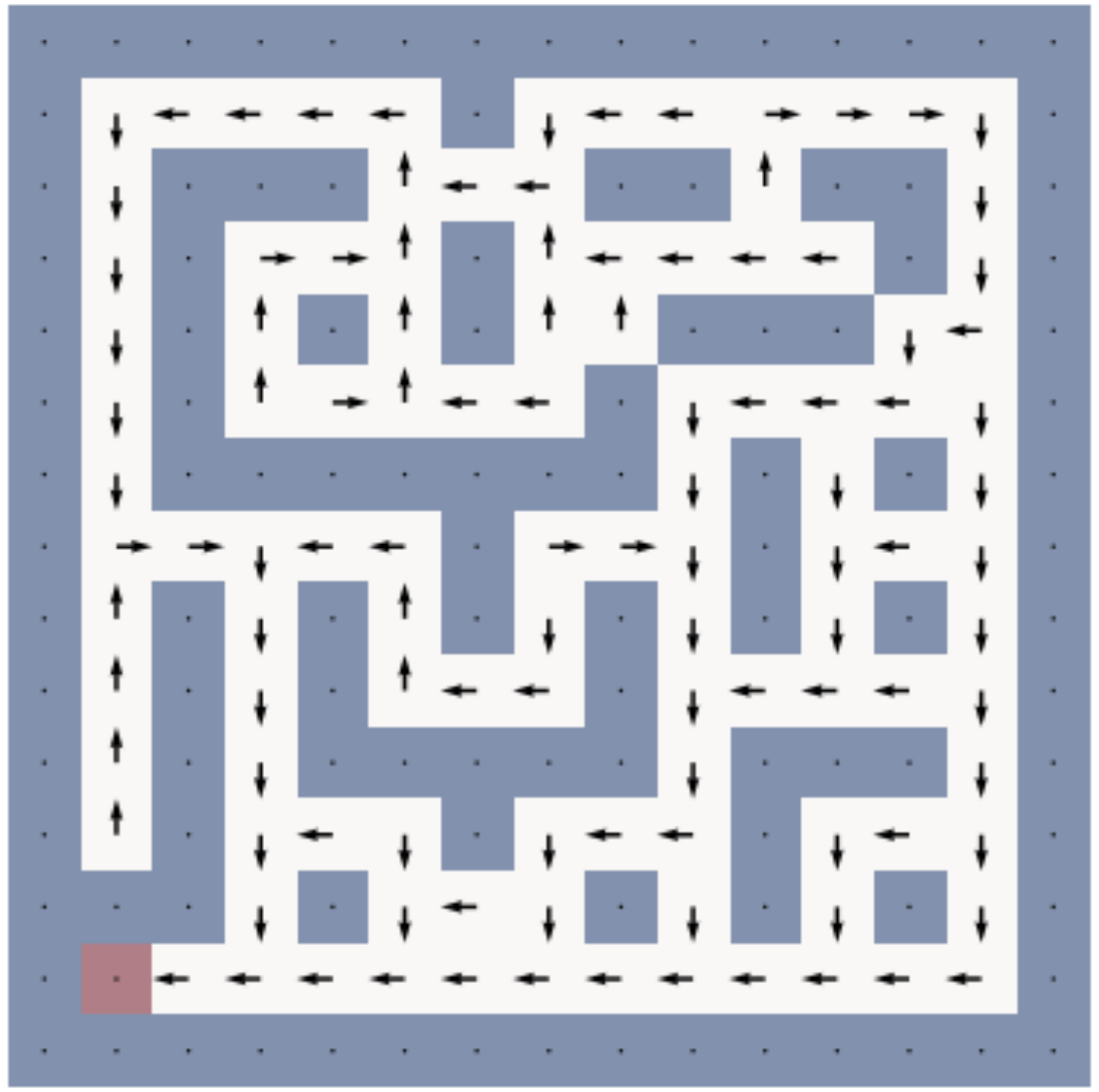
Feed in M and $\circlearrowleft 90^\circ \circ M$

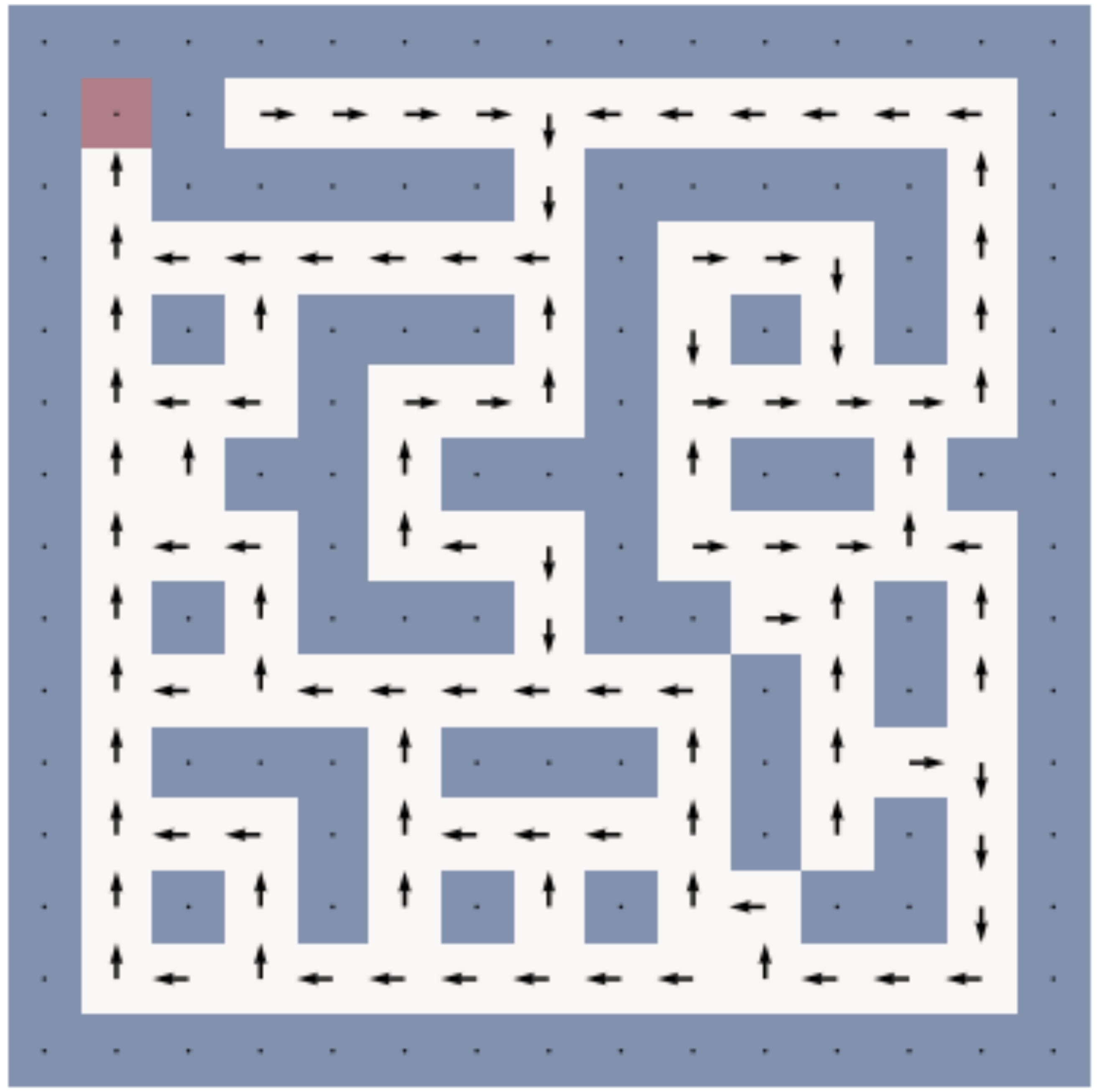


VIN output doesn't satisfy equivariance

Visualization: SymVIN

Feed in M and $\circlearrowleft 90^\circ \circ M$





SymVIN guarantees output is equivariant

Contributions

- Introduce a framework for incorporating symmetry into path-planning problems
- Prove that value iteration for path planning can be treated as a steerable CNN
- Show that Symmetric Planners improve in training efficiency and generalization

Thank you!

Check out our project website:

http://lfzhao.com/SymPlan

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