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Integrating Symmetry Into Differentiable Planning With Steerable Convolutions

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Symmetry can improve training and generalization of differentiable planning. We demonstrate on 2D path planning using Value Iteration Networks with rotation and reflection equivariance.

http://lfzhao.com/SymPlan

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1. Symmetry in Path Planning Problem

Path Planning

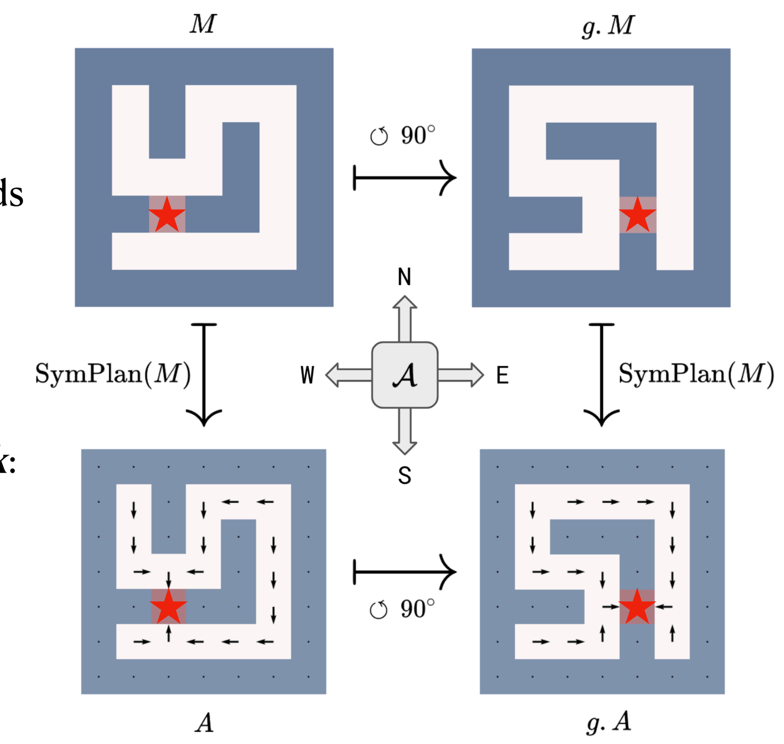
Each column is to compute the **shortest path / optimal actions** towards the red star goal location.

$\circlearrowleft 90^\circ \circ (\text{Plan}(M)) = \text{Plan}(\circlearrowleft 90^\circ \circ M)$

Equivariance in Planning

Symmetry exists in path *planning* ***task***: ***rotating input map produces rotated optimal path/actions.***

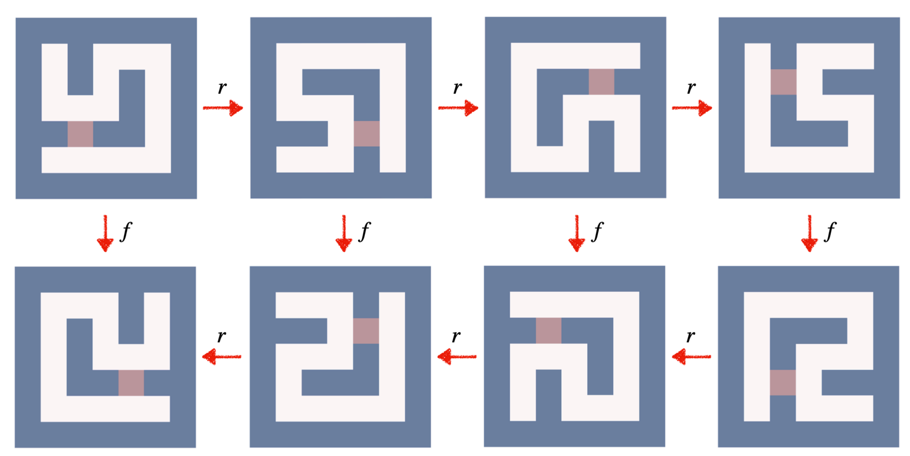
We use symmetry into *differentiable planning* ***algorithms*** by injecting ***correct equivariance constraints***.



2. Visualization of Symmetry Transformations

The symmetry of a 2D grid includes ***4 rotations times 2 reflections***, totally 8 transformations. It is called ***D_4 dihedral group***.

With equivariance, 8 states related by symmetry are effectively ***"aggregated"*** into one state, hugely ***reduce state/action space*** in planning.



3. Practice: Symmetric VIN using Steerable Convolution

Every update is equivariant — Local Equivariance

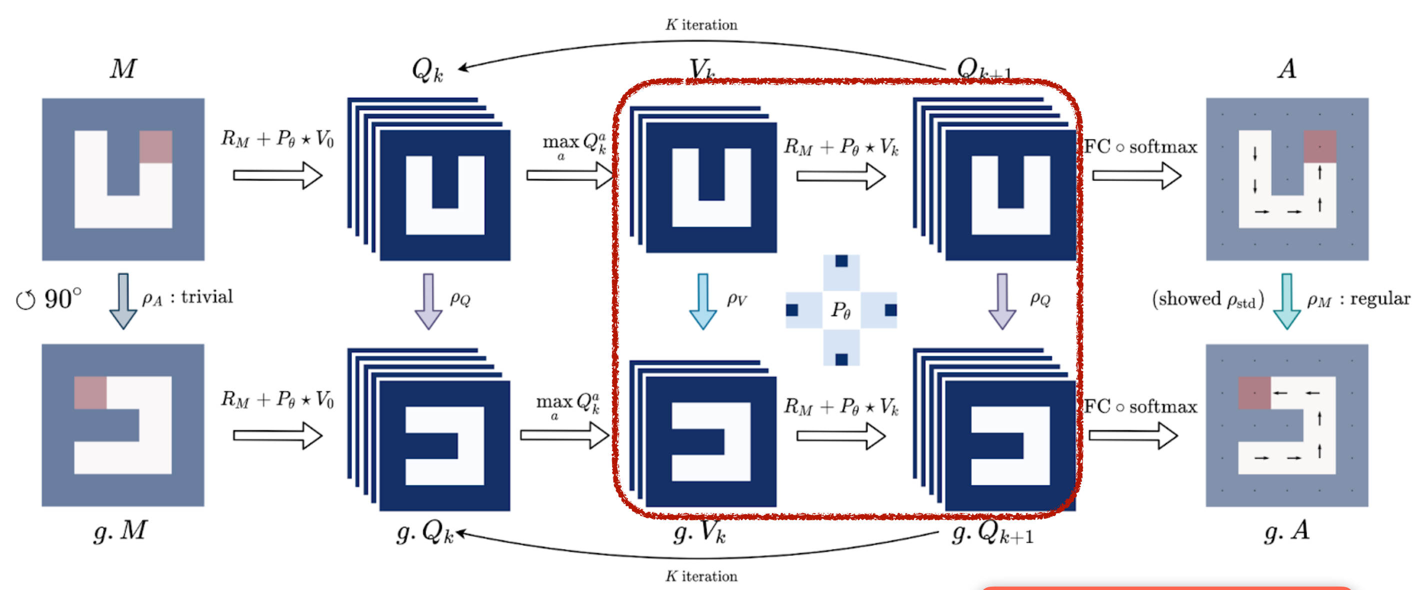
Entire planning is equivariant — Global Equivariance

$$\circlearrowleft 90^\circ \circ \mathbf{VI}(M) \equiv \circlearrowleft 90^\circ \circ \mathcal{T}^\infty[V_0] = \mathcal{T}^\infty[\circlearrowleft 90^\circ \circ V_0] \equiv \mathbf{VI}(\circlearrowleft 90^\circ \circ M)$$

High level idea: enforce equivariance constraints for each layer/step in planning (e.g. local value update), thus the entire planning procedure is globally equivariant.

Each row is a VIN process. Two rows are related by a rotation $\circlearrowleft 90^\circ$.

We consider a specific type of differentiable planning algorithm, ***Value Iteration Network (VIN)***. Each row shows a VIN that iteratively approximates the fixed point. We enforce equivariance constraints such that ***every two columns*** are equivariant: rotating the input guarantees the output is also rotated.



Every pair is equivariant

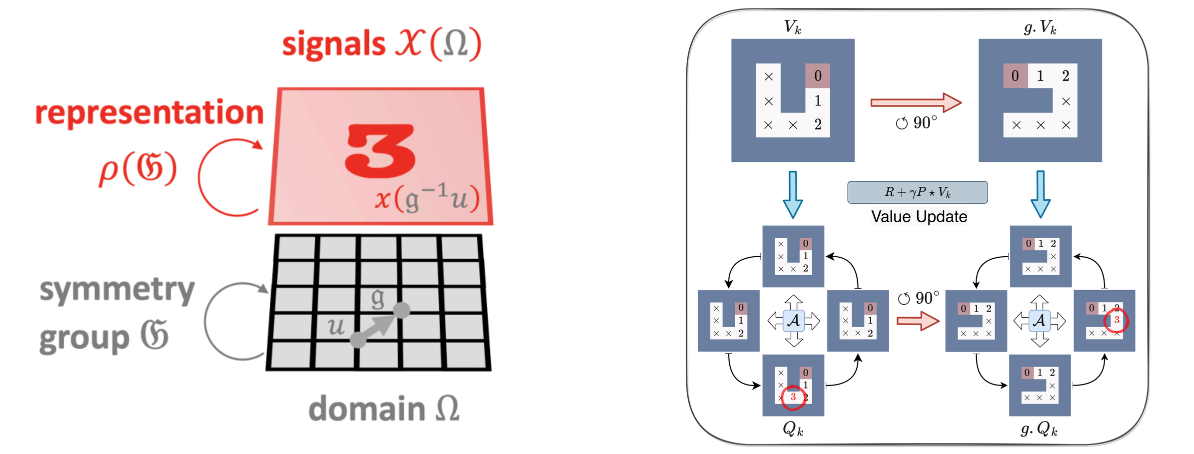
We use steerable convolutions to integrate symmetry in VINs.

Replace

$$\bar{Q}^{(k)} = \bar{R}^a + \textbf{Conv2D}(\bar{V}^{(k-1)}; W_a^V) \longrightarrow \bar{Q}_{\bar{a}}^{(k)} = \bar{R}_{\bar{a}} + \textbf{SteerableConv}(\bar{V}; W^V)$$

To enforce equivariance w.r.t. rotation and reflection for $V_k \mapsto Q_k$, we use ***steerable convolution*** layer by *replacing regular Conv2D*.

4. Insights and Theoretical Results



1: Represent (value) functions as "fields"

2: Value iteration as convolution (network)

The symmetry G comes from the ***underlying geometric space*** $\Omega = \mathbb{Z}^2$, as studied in *Geometric Deep Learning*. Value functions are fields/signals over it: $\mathbb{Z}^2 \to \mathbb{R}$. Thus, the equivariant transformations between such signals are ***equivariant (steerable) CNNs***.

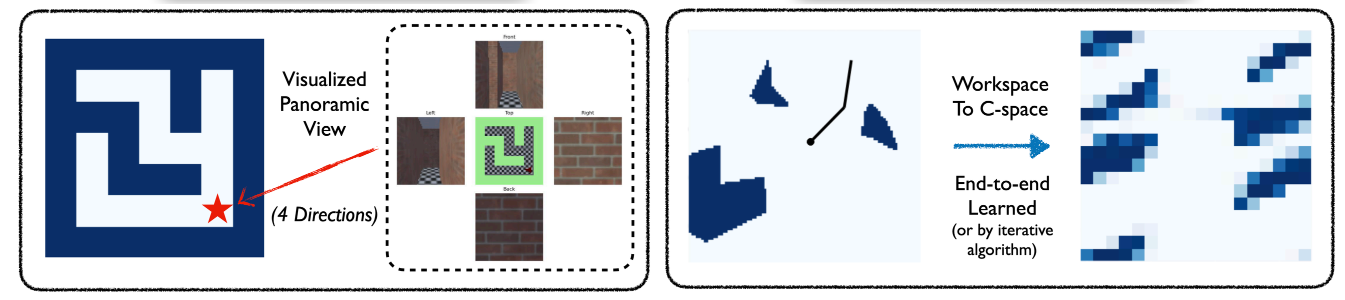
Theorem 1 (informal): **Value iteration** for **path planning*** is equivariant to translation, rotation, and reflection

Theorem 2 (informal): **Value iteration** for **path planning*** is a form of **steerable convolution network****

5. Experimental Setup and Selected Results

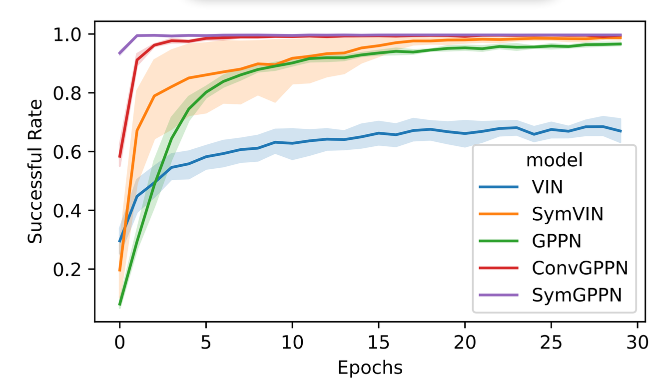
2D and Visual Maze Navigation

2-DOF Manipulation In Workspace and C-space

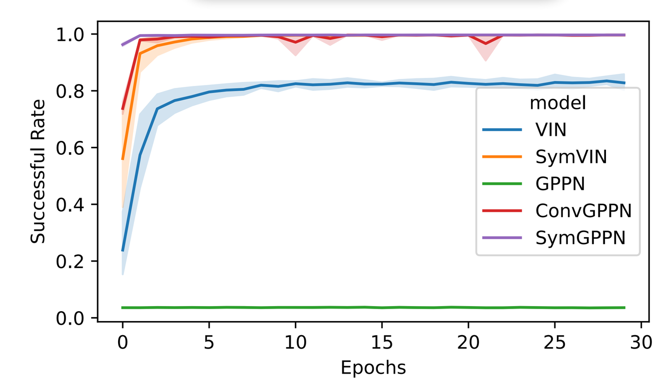


We experiment on 2D path planning in 4 different navigation and manipulation tasks. We use ***given*** 2D grid map or ***learned*** map (visual navigation and workspace manipulation, using a mapper module).

2D Maze Navigation



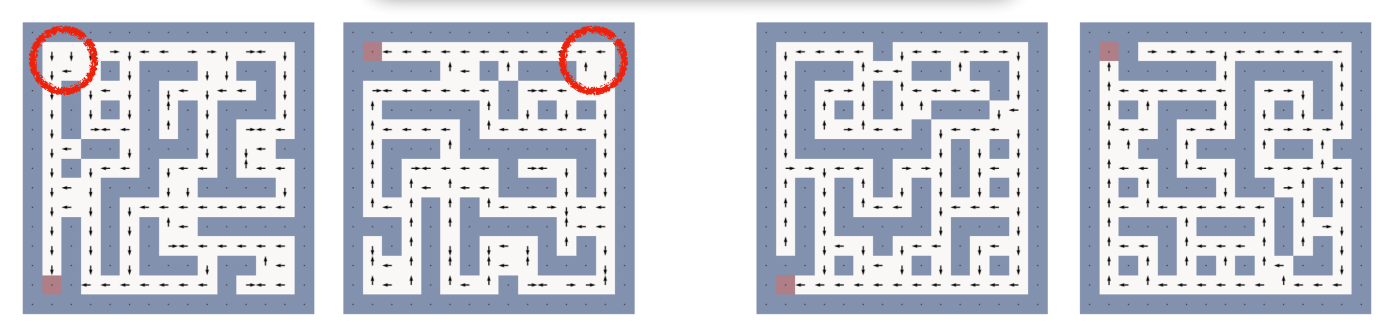
2-DOF Manipulation



SymVIN (+rotation/reflection) brings significant boost over **VIN** in ***training speed and generalization***, *similar for* ***SymGPPN (+translation/rotation/reflection)*** *vs.* ***ConvGPPN (+translation)*** *vs.* ***GPPN****, in all tasks. Figures show two tasks' training curves.*

6. Visualization of Equivariance in Planners

Feed in M and $\circlearrowleft 90^\circ \circ M$



VIN doesn't satisfy equivariance

SymVIN guarantees equivariance

Visually, we feed a map and its rotated version into VIN and SymVIN (+rotation/reflection equivariance). SymVIN **guarantees** outputs on two maps are ***exactly same under rotation***, which means the ***solution space*** is ***reduced***.